AFRL-OSR-VA-TR-2015-0051

TOPOLOGICAL QUANTUM INFORMATION IN A 3D NEUTRAL ATOM ARRAY

David Weiss
PENNSYLVANIA STATE UNIVERSITY

01/02/2015
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTB
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
12-23-2014

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
12-01-2008-9-30-2014

4. TITLE AND SUBTITLE
(DARPA) TOPOLOGICAL QUANTUM INFORMATION IN A 3D NEUTRAL ATOM ARRAY

5a. CONTRACT NUMBER
FA9550-09-1-0041

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
David Weiss

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Penn State

8. PERFORMING ORGANIZATION REPORT NUMBER
UP76XR0

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR
DARPA

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Work was performed to build core elements of a quantum computer using neutral atoms in an optical lattice, with the ultimate end to execute a version of the Kitaev toric code Hamiltonian model . Toward that end we have demonstrated the execution of single qubit gates on any arbitrary sequence of individual lattice sites in a 5×5×5 array. This entailed improving laser cooling in a 3D large spacing lattice, developing flexible state manipulation techniques, and demonstrating long atomic coherence times (exceeding 5 seconds). We designed, built and installed two MEMS mirror-controlled addressing beams that allow us to rapidly shift target atoms into resonance with microwave fields for the execution of gates. We demonstrated that we can perform single qubit gates in ~500 μs on target atoms without affecting quantum information in non-target atoms. On the theoretical side, we developed a paradigm for implementing digital quantum simulations in finite systems, including a dissipative mechanism that allows thermalization to arbitrary temperatures, and applied this to realization of the toric code Hamiltonian within our trapped neutral atoms architecture.

15. SUBJECT TERMS
quantum computing, ultracold atoms

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** David Weiss
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			**19b. TELEPHONE NUMBER** *(include area code)* 814-861-3578

DARPA Final Report
Grant # FA9550-09-1-0041
December 2014

Technical Area
Quantum Information

Lead Organization
Penn State

Team Members
Prof. David Weiss, Penn State
Prof. Birgitta Whaley, University of California, Berkeley
Prof. Jungsang Kim, AQT/Duke University

Proposal Title
TOPOLOGICAL QUANTUM INFORMATION IN A 3D NEUTRAL ATOM ARRAY

Research Areas
Fundamentals of Quantum Information
Science and Technology

Technical Point of Contact
Prof. David S. Weiss
Professor and Associate Head, Dept. of Physics
Pennsylvania State University,
University Park, PA 16802
Ph. 814-863-3076, dsweiss@phys.psu.edu

Abstract

Experimental and theoretical work was performed with the goal of developing the system of neutral atoms in a 3D optical lattice into a flexible platform for quantum information. The effort was specifically directed toward implementing the Kitaev toric code Hamiltonian model, but accomplishing that goal requires building a universal quantum computer with at least ~25 qubits. Toward that end we have demonstrated the execution of single qubit gates on any arbitrary sequence of individual lattice sites in a 5×5×5 array. This entailed improving laser cooling in a 3D large spacing lattice, developing flexible state manipulation techniques, and demonstrating long atomic coherence times (exceeding 5 seconds). We designed, built and installed two MEMS mirror-controlled addressing beams that allow us to rapidly shift target atoms into resonance with microwave fields for the execution of gates. We demonstrated that we can perform single qubit gates in ~500 μs on target atoms without affecting quantum information in non-target atoms. On the theoretical side, we developed a paradigm for implementing digital quantum simulations in finite systems, including a dissipative mechanism that allows thermalization to arbitrary temperatures, and applied this to realization of the toric code Hamiltonian within our trapped neutral atoms architecture.

We have made significant progress on all the remaining stages along the path of turning the system into a universal quantum computer, and thus implementing the Kitaev model. The next step is perfectly filling the lattice, which requires the demonstrated single qubit technology, the rapid and perfect determination of site occupancy, which we have demonstrated, and implementation of state-dependent site translations, which are integral to our cooling procedure. The penultimate step will be entanglement of nearby atoms, which requires the overlapping of two each of two color MEMS controlled beams, which technology is nearly complete. It also requires narrow-linewidth Rydberg excitation lasers, the development of which is in progress. Theoretical optimization of entangling operations in the presence of experimental noise is critical to the success of this venture, and this has now been done using methods of coherent control. The final step will be executing all these advances at the same time in the experiment.

Overview of Accomplishments

The broad goal of this work was to develop a neutral atom 3D optical lattice system into a quantum computer sufficient to entangle >25 qubits and execute a large number of quantum gates with them, presumably on the order to 1000, before appreciable decoherence occurs. The atom targeting was to be done with microwaves and with laser beams that reflect from microelectromechanical systems (MEMS) mirrors, which would be built for this purpose. The merging of the 3D optical lattice with single qubit addressing beams has been accomplished. Along with a series of technical developments in our neutral atom system, this has enabled us to demonstrate arbitrary single qubit gates on atoms at any site within a 5×5×5 array. With typical site occupancy of 40%, that means we have 50 qubits at entanglable distances from each other that can be independently coherently addressed, which is about a factor of three more than the next best system in this regard, trapped ions. The coherence time exceeds the single qubit gate time by a factor of more than 10,000, and we expect shorter entangling gates, as well as future speed-up o the single qubit gates. But more needs to be accomplished to catch up to ion systems, since we have not yet reached our goal of converting initial random site occupancy to perfect site occupancy and we have not yet incorporated MEMS-based addressing beams for executing Rydberg entangling operations. We will now enumerate and illustrate in more detail the accomplishments enabled by this grant.

Neutral atoms in 3D optical lattices

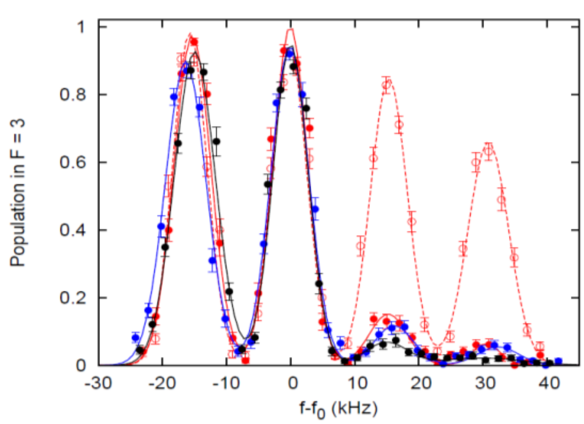


Figure 1: Projection sideband cooling. The pink curves show the (from left to right) Δn=+1,0,-1 and-2 transitions after PGC. The reduced size of the Δn=-1 peak relative to Δn=+1 is indicative of lower temperatures. The effectiveness of projection cooling is illustrated by the significantly decreased height of the Δn=-1 sidebands in each of the 3 orthogonal directions.

Projection cooling: We developed a new method for laser cooling atoms in optical lattices, which overcomes most of the difficulty associated with being only weakly in the Lamb-Dicke

limit. We have demonstrated cooling of 76% of the atoms into the 3D vibrational ground state of their lattice site (see Fig. 1).

Automated site maps: We have developed the software to rapidly convert a stack of 5 images from 5 planes in the 3D optical lattice into a reliable site occupancy map, with negligible error (see Fig. 2). Combined with state-selective clearing of atoms, this allows us to, in parallel, measure the quantum states of all atoms in the array.

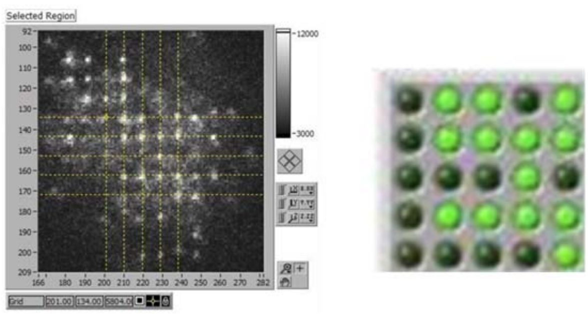


Figure 2: Site occupancy identification. We take an image for each lattice plane, and then fit the intensity pattern at each site, for a preliminary occupancy map. We then refine the map by using information from the adjacent out-of-focus planes to improve the fits. The sequence converges into a unique stable pattern in one iteration.

Coherence times: We have demonstrated coherence times in our qubit states of better than 7 s (see Fig.3), which is significantly longer than other trapped neutral atom systems that do no employ additional large magnetic fields, which would compromise gate operations. The improvement stems from our low temperatures.

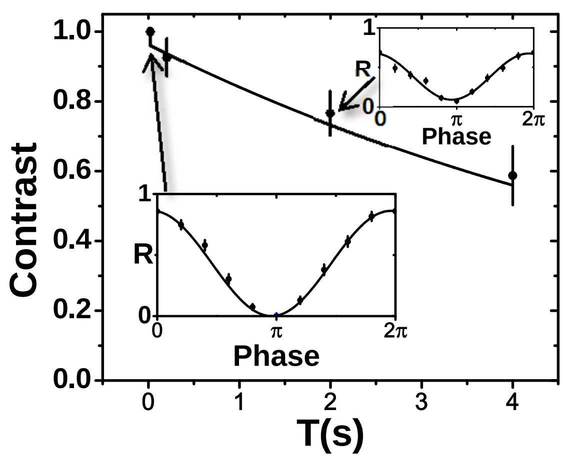


Fig. 3: Fringe contrast for a spin-echo sequence with all the atoms in the lattice acted on in parallel without addressing. The fit exponential time constant is 7.4 s. Since the contrast is lost due to spontaneous emission, the rate of which depends on an atom's vibrational state, the true function is more complicated.The insets are the fringes at the indicated times, where the phase of the final $\pi/2$ pulse in a $\pi/2$ π-$\pi/2$ is varied.

Beam stabilization: We have superlative temperature control in the lab (~30 mK rms near the atoms) but there are still slow thermal gradient drifts over the course of the day. In order to achieve reliable site addressing, we have had to develop several specialized nested locking loops. The phases of the three lattice beam pairs are adjusted with a very slow servo so that the lattice centers do not move by more than 150 nm in any direction, which we measure by more image processing as in Fig. 2. The addressing beams intensity near the atoms is servo-controlled, for

stability and also to account for site-dependent differences. The pointing of the addressing beams is measured on position sensitive detectors, and they are stabilized to 150 nm.

MEMS addressing*:* We have incorporated two MEMS mirror-directed beams, which propagate through two five-lens optical transfer systems into our optical lattice apparatus. By reproducibly crossing them at target sites (see Fig. 4), we can shift only the target atoms into microwave resonance, as illustrated in Fig. 5.

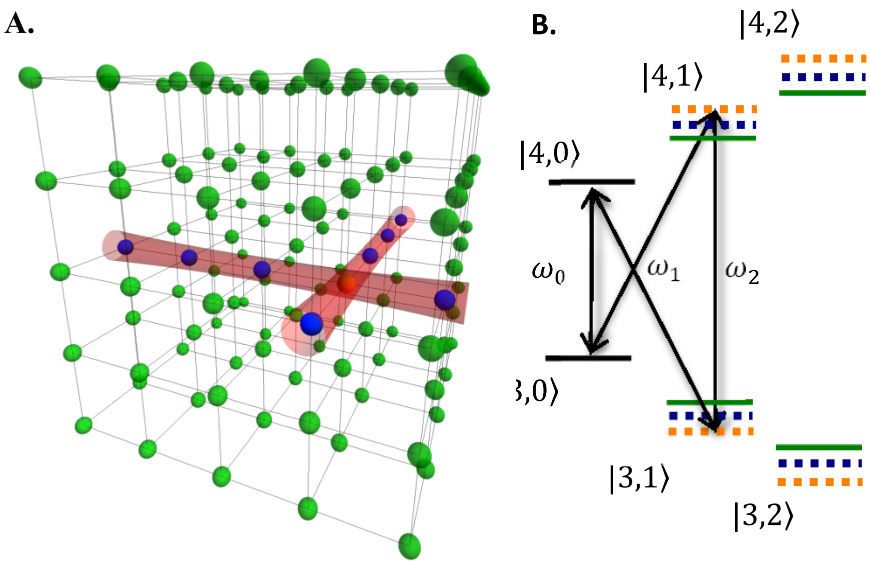
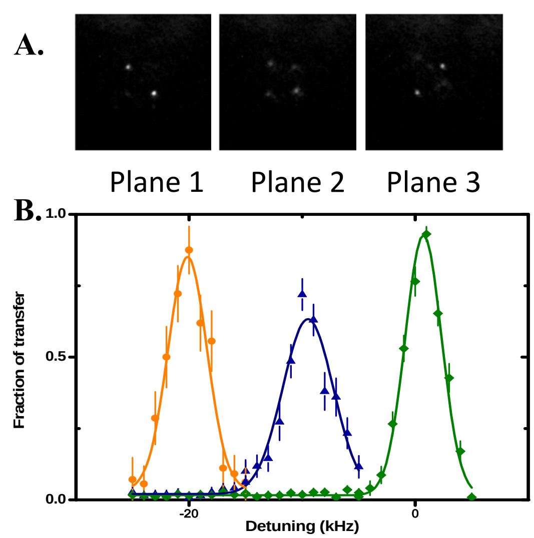


Figure 4: Addressing atoms in a 5×5×5 array. A.) Two circularly polarized laser beams pass through the lattice. The target atom sees twice the intensity of any other atom. The B-field is in the addressing beam plane at 45^0 to the beams. B.) The addressing transitions in the Cs ground state hyperfine manifold (only partially shown here). Only the target atom is ac Stark shifted into resonance with the microwaves. It is tranferred to the m$_F$=1 states, in which an arbitrary rotation on the Bloch sphere can be made. The target atom is then returned to the qubit states.

Figure 5: Addressing single lattice sites. A.) The sum of images of F=3 atoms in 20 implementations after a four Blackman microwave pulse sequence transfers two targeted atoms in two planes (1 and 3) from F=4,m$_F$=0 to F=3,m$_F$=-1. Non-targeted atoms, which occupy all the other sites, do not make the transition. The Plane 2 image shows the out-of-focus atoms in the adjacent planes. B.) The fraction of atoms transferred to the temporary qubit state as a function of the microwave frequency. The green points are from sites that are not crossed by the addressing beams, the blue points are from sites that are crossed by one addressing beam, and the orange points are from target atoms. The targeting microwave frequency is the orange peak.



Single qubit gates*:* To execute single qubit gates, we employ the sequence illustrated in Fig. 4B, within a spin-echo manifold of microwave pulses that cancel any net effect on non-target atoms (see Fig. 6). The results are illustrated in Fig. 7, which shows the execution on two non-coplanar

target atoms of a π-phase gate (Fig 7A), a π/2-phase gate (Fig. 7B), and a π/2-amplitude gate (Fig. 7C). Since we can independently address 125 lattice sites in this way, this is by far the largest single system in which single qubits can be independently addressed. As we will discuss, converting this capability into a comparably large entangled system requires more work.

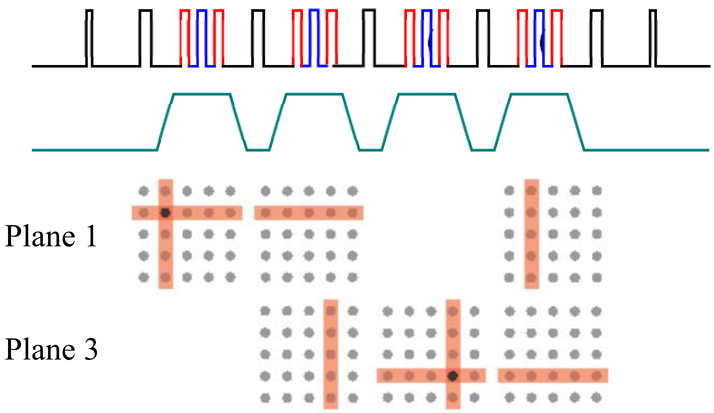


Figure 6: Microwave pulse sequence for single qubit gates. The top line schematically shows the microwave pulses as a function of time, where ω_0 is black, ω_1 is red, and ω_2 is blue. The second line shows the intensity of the two addressing beams. The final two lines show the spatial location of the two addressing beams during each gate. The first and third gate periods address a single atom in Plane 1 and 3 respectively. The second and third gates do not address any atom, but serve to cancel unwanted phase shifts at non-target atoms.

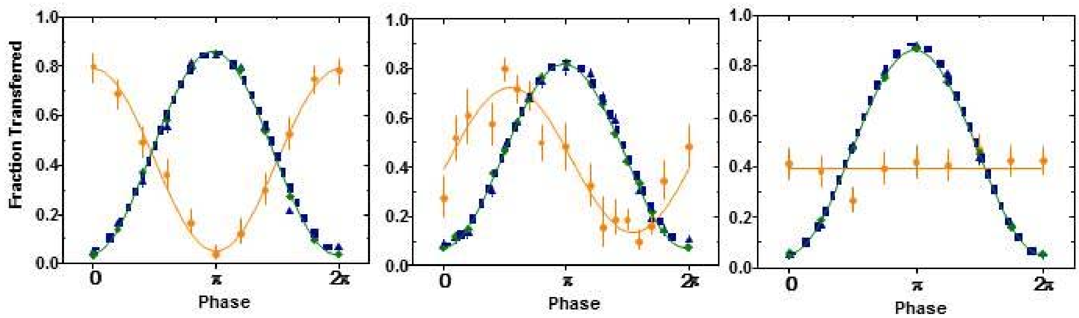


Figure 7: Targeted single qubit gates. A.) The orange circles are the fringes of two targeted atoms with a π-phase gate (see Figure. 7). B.) C.) The interference fringe from two atoms targeted with a π/2-amplitude gate. For all figures, the green diamonds correspond to atoms that are not in line with any addressing beam and the blue triangles correspond to non-targeted atoms in the line of a single addressing beam. Neither is ever significantly shifted. The size of the error bars in all cases is dominated by counting noise.

<u>MEMS mirrors for optical lattice addressing</u>

Phase I MEMS beam steering system: The goal in this part of our project was to construct and integrate three optical systems capable of projecting multiple beams of control laser beams (at 870-880nm range, 1063nm and 455nm, respectively) necessary for the qubit gates onto the 3D lattice of atoms. In Phase I, we designed optical systems capable of such 3D addressing, of up to six beams. Figure 8 shows the result of the initial efforts. A fiber array was fabricated, and methods to attach the microlens array on the front was developed that created arrays of

collimated beams to be projected onto the MEMS mirror array. After the beams go through the MEMS beam steering system, we utilized a lenslet array with different focal lengths inserted in the projection optics that allowed us to focus the beams in different focal planes at the atomic array (shown in Fig. 8f). Custom MEMS mirrors were developed using Sandia's SUMMiT V fabrication process, where fast switching speeds of <10μs was realized.

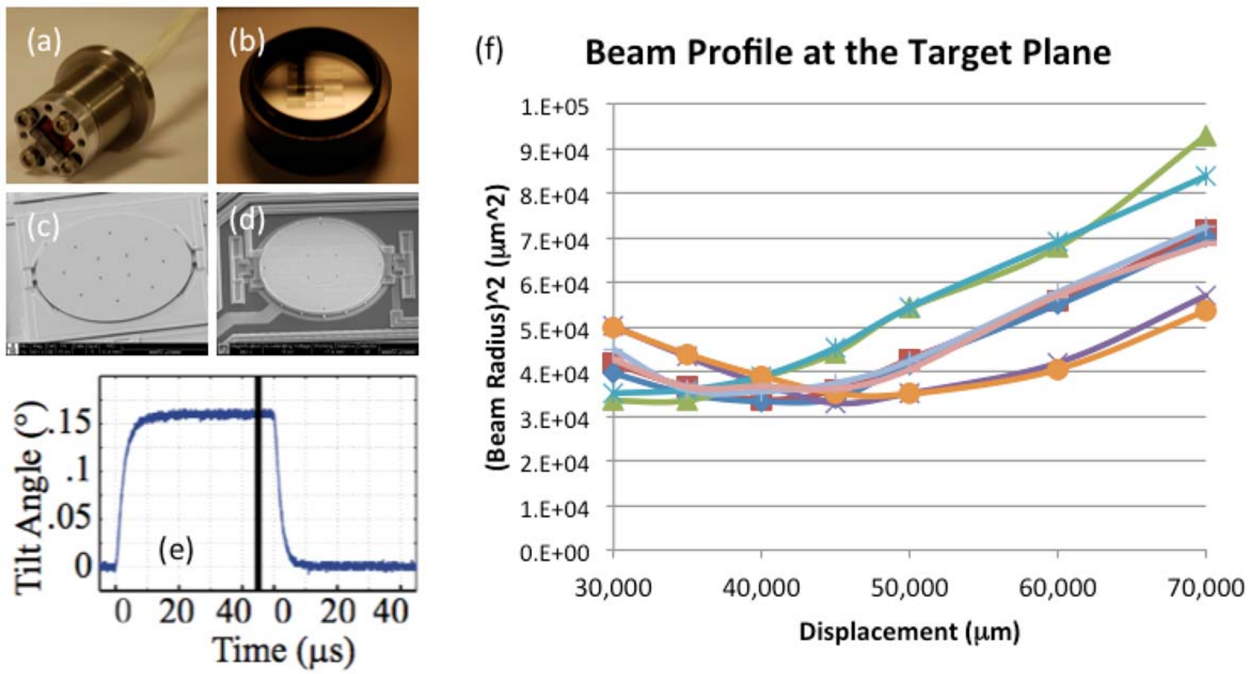


Figure 8: Components and performance of the initial 3D beam steering system. (a) Fiber array aligned with microlens array. (b) Lenslet array with different focal lengths that enables shift in focal points. (c, d) MEMS mirrors fabricated using the Sandia SUMMiT V process. (e) Switching speed for the initial MEMS mirrors. (f) Demonstration of the 3D focusing of the beams using the lenslet array.

Upon further optics simulations, we concluded that this method of performing 3D steering works well if there is sufficient numerical aperture (NA) in the projection optics to bring in several beams to the atomic system. Since the 3D optical lattice requires large NA (0.55) to tightly focus the addressing beams, bringing beams through multiple spatial modes in the projection optics was not deemed adequate (NA is the scarce resource in the optical design space). We concluded that we would focus on 2D steering in one target plane, bringing the target atomic qubits to this "addressing plane," where multiple beams are multiplexed through polarization and/or beamsplitters.

Phase II MEMS beam steering system: Our Phase II system development focused on three main tasks, that include (1) Optimization of MEMS devices for adequate steering range and speed, (2) packaging of the devices for long-term, stable operation, and (3) design and implementation of the optical systems and control interfaces for integration into the atomic lattice experiment. The goal was to construct three optical systems at the three target wavelengths and deliver them from AQT to Penn State for system integration demonstration. The optical system was carefully modeled using commercial ray-tracing software (Zemax) before construction.

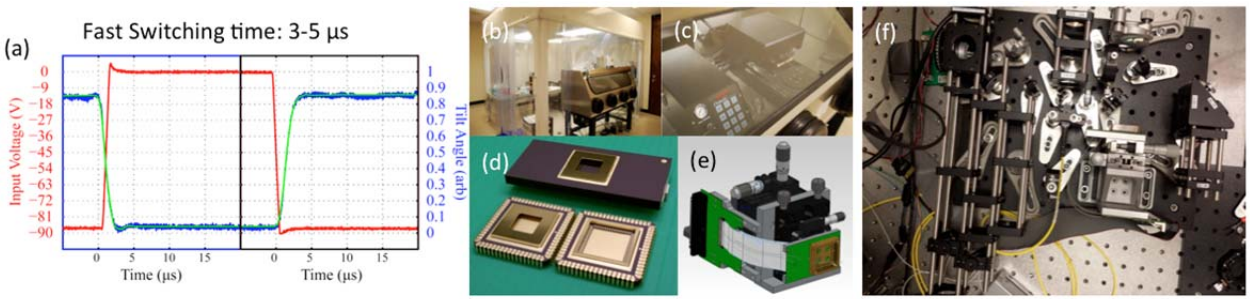


Figure 9: Progress made in Phase II of the project. (a) Optimized MEMS devices feature < 5μs switching speeds. (b) Small cleanroom and (c) packaging facility established at AQT for reliable packaging of the MEMS devices. (d) Some customized ceramic packages designed to hold the MEMS devices. (e) Compact optomechanical design for holding the MEMS devices for the system integration. (f) Final system assembled for a two-beam steering system. The system within a 12" x 12" footprint includes the MEMS beam steering system and first stage of projection optics that incorporates a 2x2 beamsplitter that combines the two beams into a single spatial mode.

Figure 9 shows the progress made in Phase II. After two iterations of device fabrication, the performance of the MEMS devices were improved to obtain switching times on the order of 3μs (Fig. 9a), with adequate angular range to achieve steering over 5x5 lattice sites. The packaging capability was established that allows us to enclose the MEMS devices in an ultra-dry inert environment that will ensure long-term stable operation (Fig. 9b-d). We also tested the mechanical stability of the packaged MEMS mirrors, and concluded that this device features mechanical stability limited by the fundamental thermal-mechanical noise, and therefore is sufficient to provide interferometric stability to the steered beams if necessary. A compact optomechanical system was designed to position the MEMS devices in the beam steering system that provides all necessary degrees of freedom for alignment, yet removing unnecessary motion for optimal stability (Fig. 9e). Figure 9f shows the assembly of the final system. It consists of a compact fiber array generating two independent beams, which is steered in the MEMS system.

The package (with footprint of less than 12 inches on a side) also includes the first stage of projection optics that reduces the beam size by a factor of 10. We have incorporated a 2x2 beamsplitter in the middle of this projection optics, which combines the two beams into a single spatial mode for optical projection onto the atoms in the final stage. All three systems have been constructed as of December 2014, and two systems (870nm and 455nm) have been delivered to Penn State from AQT. The final system (1063nm) will be delivered in early 2015 to complete the project.

Theory

Stroboscopic realization of n-body interactions: We developed a stroboscopic scheme for realization of a Hamiltonian with n-body interactions on a set of neutral atoms trapped in an addressable optical lattice, using only 1- and 2-body physical operations, together with a dissipative mechanism that allows thermalization to finite temperature or cooling to the ground state. We demonstrated the scheme with application to Kitaev's toric code Hamiltonian, showing that the required 4-body Hamiltonian terms can be generated from 2-body interactions with errors small enough that the toric code ground state can be faithfully simulated. Using a serial sequence of Rydberg gates and 1-qubit rotations, we estimated that a gap of Δ~30nK can be achieved in a minimal lattice of 18 atoms. We also developed a method for cooling of the trapped atom system during simulation of the toric code Hamiltonian to remove entropy/errors generated by imperfect gates. In this scheme, dissipative control of ancilla atoms coupled to the simulation atoms allows for either cooling towards the ground state or thermalization to a finite temperature state. Effective temperatures below the gap were predicted to be achievable provided that individual 2-qubit gates can be realized with a composite error probability of 10^{-4} or less. We developed explicit optimal gate sequences for the required 2-body unitaries.

Quantum simulation of general stabilizer Hamiltonians: We then extended this stroboscopic approach to robust finite temperature quantum simulation of general stabilizer Hamiltonians, assuming again realization in a physical system consisting of a finite set of neutral atoms trapped in an addressable optical lattice that are controllable via one- and two-body operations together with dissipative one-body operations such as optical pumping. We showed that these minimal physical constraints suffice for design of a quantum simulation scheme for any stabilizer

Hamiltonian at arbitrary temperature and demonstrated the approach with application to both the Abelian and non-Abelian versions of the toric code.

***Coherent control for high fidelity gates required for stroboscopic simulation**:* To map the stroboscopic scheme onto the experimental implementation in the Weiss/Kim groups, we made detailed calculations of the elemental two-body Rydberg gates that constitute the core of our stroboscopic scheme for realizing the toric code. Controlled-PHASE (CPHASE) gates can be realized with trapped neutral atoms by making use of the Rydberg blockade. Achieving the ultrahigh fidelities required for quantum computation with such Rydberg gates, however, is compromised by experimental inaccuracies in pulse amplitudes and timings, as well as by stray fields that cause fluctuations of the Rydberg levels. We therefore undertook a comparative study of analytic and numerical pulse sequences for the Rydberg CPHASE gate that specifically examines the robustness of the gate fidelity with respect to such experimental perturbations as experienced in the Weiss laboratory at Penn State. Analytical pulse sequences of both simultaneous pulses in the idealized three pulse protocol of Jaksch, Cirac and Zoller (2000) were compared with a stimulated Raman adiabatic passage (STIRAP) implementation. Both of these analytic approaches to the Rydberg CPHASE gate were found to be at best moderately robust under such experimental perturbations. In contrast, optimal control theory was seen to allow generation of numerical pulses that are inherently robust within a predefined tolerance window. The resulting numerical pulse shapes display simple modulation patterns and could be rationalized in terms of an interference between distinct two-photon Rydberg excitation pathways. Pulses of such low complexity should be experimentally feasible, allowing gate fidelities of order 99.90–99.99% to be achievable under realistic experimental conditions.

***Detailed analysis of thermalization dynamics for topological qubits in finite systems**:* Motivated by our analysis of entropy removal from a finite simulation system via thermalization with a dissipatively driven ancilla system, we made an analysis of the relaxation dynamics of finite-size topological qubits in contact with a thermal bath. Using a continuous-time Monte Carlo method, we explicitly computed the low-temperature nonequilibrium dynamics of the toric code on finite lattices. In contrast to the size-independent bound predicted for the toric code in the thermodynamic limit, for finite lattices such as in the Penn State/AQW experiments, we identified a low-temperature regime below a size-dependent crossover temperature, in which

there is nontrivial finite-size and temperature scaling of the relaxation time. We demonstrated that this nontrivial finite-size scaling is governed by the scaling of topologically nontrivial two-dimensional classical random walks. The transition out of the low-temperature regime was found to define a *dynamical* finite-size crossover temperature that scales inversely with the log of the system size, in agreement with a crossover temperature defined from equilibrium properties. We found that both the finite-size and finite-temperature scaling are stronger in the low-temperature regime than above the crossover temperature. Since this finite-temperature scaling competes with the scaling of the robustness to unitary perturbations, this analysis provides insight for the scaling of memory lifetimes of various possible physical realizations of topological qubits in finite size lattices.

Impact of theoretical work on the field of Quantum Science and Technology: The theory work on realization of the toric code Hamiltonian with trapped neutral atoms has several key impacts to the field of quantum science and technology. The stroboscopic scheme and its thermalization have generated a powerful paradigm for implementing digital quantum simulations in finite systems, which was subsequently extended to general stabilizer Hamiltonians and the non-Abelian toric code. The pulse sequences that were subsequently generated by methods of coherent control provide readily implementable schemes for high fidelity gates that are robust to imperfections induced by experimental constraints. Finally, the recent analysis of relaxation dynamics identified a low temperature regime for finite sized experimentally realizable systems in which the optimum balance between robustness to unitary perturbations and thermal robustness is achieved.

Current status and partially completed goals

The three incomplete goals on this project are: obtaining perfect filling of lattice sites; incorporating Rydberg entangling gate operations; and executing Kitaev's toric code.

All the elements of filling lattices have been developed, including rapid site occupancy determination, state-dependent lattice shifting, and single site addressing. The hardware and software needed to adjust the sequence of control for the MEMS mirrors while the atoms are in the lattice has also been developed, but has not yet been tested with the atoms.

The entangling Rydberg gates requires pulsed narrow linewidth laser beams at two disparate wavelengths (1063 nm and 455 nm), which have been partially developed, but the execution is not complete. It also requires four more MEMS mirror systems, all integrated to approach the atoms from the same direction. The necessary achromatic imaging lens has been designed and built. MEMS mirrors for the necessary wavelengths are nearly fully developed, as is the optical system for combining all the beams. Our expectation is that, because our atoms are much colder and better spatially localized than in previous work (<200 nm), we will be able to achieve high fidelity with these gates.

Execution of Kitaev's code must wait until all the experimental elements are complete. But the theoretical underpinnings of this execution have been developed. In particular, it has been demonstrated that n-body interactions can be implemented stroboscopically, which is the theoretical proof of principle of the concept. Also, significant theoretical work has improved the prospect for robustness in all the required gate executions.

Publications

1. C. M. Herdman, Kevin C. Young, V. Scarola, M. Sarovar, K. B. Whaley, "Stroboscopic Generation of Topological Protection", Phys. Rev. Lett. **104**, 230501 (2010).

2. X. Li, T.A. Corcovilos, Y. Wang and D.S. Weiss, "3D projection sideband cooling", *Phys. Rev. Lett.,* **108**, 103001 (2012).

3. K. C. Young, M. Sarovar, J. Aytac, C. M. Herdman and K. B. Whaley, "Finite temperature quantum simulation of stabilizer Hamiltonians", J. Phys. B: At. Mol. Opt. Phys. **45** 154012 (2012).

4. C. Knoernschild, T. Kim, P. Maunz, S. G. Crain, and J. Kim, "Stable optical phase modulation with micromirrors," Opt. Express 20, 3261 (2012).

5. M. H. Goerz, E. J. Halperin, J. M. Aytac, C. P. Koch and K. B. Whaley, "Robustness of high-fidelity Rydberg gates with single-site addressability", Phys. Rev. A **90**, 032329 (2014) (Editor's Choice)

6. C. Daniel Freeman, C. M. Herdman, Dylan J Gorman, K. B. Whaley, "Relaxation time dynamics of the toric code in contact with a thermal reservoir: finite-size scaling in a low temperature crossover regime", Phys. Rev. B **90**, 134302 (2014).

7. Y. Wang, X. Zhang, T. Corcovilos, A. Kumar, and D.S. Weiss, "Arbitrary single qubit gates on individual neutral atoms in a 3D optical lattice", submitted.

Ph. D. Theses:

1. **Kevin Young** “Controlling Quantum Systems for Quantum Information Processing” (Ph. D., Department of Physics) Fall 2010
2. **Christopher Herdman** “Loop Condensation in Quantum Dimer Models” (Ph. D., Department of Physics) Fall 2011
3. **Alexander Selem** “Topics in Topologically Ordered Phases of Matter” (Ph. D., Department of Physics) Spring 2013.

Personnel trained: In addition to the 3 Berkeley theory graduate students listed above who obtained their PhDs at least in part for work on this project, **Yang Wang** and **Aishwarya Kumar** at Penn State have worked entirely on this project, and **Michael Goerz** from the University of Kassel collaborated with the Berkeley group on the gate optimization work. The theoretical work on gate optimization also trained undergraduate **Eli Halperin** from Wesleyan University and Berkeley graduate student **Jon Aytac**, while the theory on dynamics of thermalization also trained Berkeley undergraduate **Dylan Gorman** and Berkeley graduate student **Daniel Freeman**. The project has trained a number of postdocs. At Berkeley these include **Vito Scarola** (now a professor at Virginia Tech), **Christopher Herdman** (now a postdoc at University of Waterloo, Canada) and **Alexander Selem** (now at Travelers Insurance). At Penn State these include: **Xiao Li** (now at NRL), **Theodore Corcovilos** (now a professor at Dusquene University) and **Xianli Zhang** (now at Microsemi Corp.)..